Getting to the Core

Algebra I – Unit 3

Equations & Inequalities In One Variable

Updated on May 8, 2013

Student Name _____ **Period** _____

This page was intentionally left blank.

Unit 3 – Equations & Inequalities In One Variable
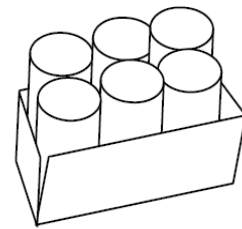# Table of Contents

This page was intentionally left blank.

Diminishing Return

Level A:

Some classes are going out for a picnic lunch. The teachers bought drinks in packs for their classes.

Thirty-three students are in Mrs. Browne's class. Mrs. Browne bought six-packs for her class. She needs helpers, so she picks students to carry one six-pack each.

Twenty-two students are in Mrs. Robinson's class. Mrs. Robinson bought four-packs for her class. She needs helpers, so she picks students to carry one four-pack each.
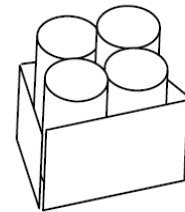
Which teacher had to pick more helpers?

Show how you found your answer.

Level B:

Amusement Center Store

Yo Yo $1.22 Doll $2.75 Duckie $1.85 Tractor $5.97 Airplane $6.47

Ball $2.16 Racecar $7.13 Dog $4.57 Jump Rope $1.46 Car $5.18

Elephant $3.16 Bear $4.89 Xylophone $7.11 Tank $6.45 Checkers $4.77

Boat $8.04 Train $6.71 Jacks $2.31 Truck $6.21 Whistle 98¢ Pinwheel 87¢

Mia has earned $43.94 of tokens playing games at the amusement center. The store in the amusement center has the following toys for sale. She plans to get toys and donate them to a local charity for needy children. The tokens are only good in this store, so she plans to spend all the tokens. What combinations of toys can she buy in order to spend all the tokens?

Show how you found your solution.

SVMI Problem of the Month   Diminishing Return   Page 2
(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@noycefdn.org.

6

Is your solution the only possible answer? Explain.

Level C:

Maxine and Sammie have the same size lawn. Maxine can mow the lawn in 24 minutes and Sammie can mow the lawn in 36 minutes. At what time will Sammie have twice as much lawn to mow as Maxine?

Maxine and Sammie have to also mow their parking strips that are the same size. Maxine can mow the parking strip in 6 minutes and Sammie can mow the parking strip in 9 minutes. At what time will Sammie have twice as much grass to mow as Maxine?

Level D:

Rollie was successful in losing weight. He had a goal weight in mind. He went on a diet for three months. Each month, he would lose one-third of the difference between his current weight and his goal weight and an additional three pounds. At the end of three months, he was just 3 pounds over his goal weight. How many pounds did he lose in those three months?

Explain how you arrived at your solution.

SVMI Problem of the Month   Diminishing Return   Page 4
(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@noycefdn.org.

8

3a-1 Understanding inequalities

1. Javier has enough to pay for his lunch at the amusement park. Lunch costs $10. What are the possible amounts that he may have? Explain your answer.

_____

_____

2. Ms. G wants to take her Avid class to Knott's for their end of the year field trip. It will only happen if more than 20 students pass the math test. How many students need to pass?

_____

_____

3. There has to be at least 20 students but no more than 60 on the bus. How many students can fill one bus? Explain your answer.

_____

_____

4. Each log on the log ride fits no more than five people. If you have less than five people, then the ride will not go. How many riders must be on log? Explain your answer.

_____

_____

Given

= "is equal to"
> "is greater than"
< "is less than"
≤ "is less than or equal to"
≥ "is greater than or equal to"

The **equal symbol** is used to show that two quantities are the same.

The **inequality symbols** are used to represent two quantities that are not necessarily equal.

For example, the inequality

$$x > 5$$

could be used to represent the sentence "John has more than $5"

Use the given symbols above and write mathematical representations for problems 1 – 4.

1. _____

2. _____

3. _____

4. _____

Let's practice!

Problem	Equation/Inequality *ex. $x > 8$*	Verbal expression *ex. "x is is greater than 8"*
There are a total of 80 students going to Knott's.		
Maritza has less than $18 for lunch and amusement.		
John wants to buy a souvenir for himself. The souvenirs cost $6 or more but less than $30.		
You must be at least 48" to ride most rides.		
Eric is having a birthday party and wants to give out party favors. He has $80 to spend on party favors.		
	$n \geq 20$	
		x is less than or equal to 50.
	$x < 10$	

Today's main idea (share your thoughts with your team!)

_____

_____

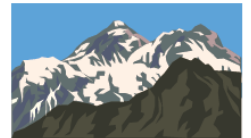**Preparing The Learner Lesson – Algebra 1**

Part 1: Matching Exercise

Scenarios:	Scales:
A: Michael is counting the number of fish in a bowl.	1: Count by thousands
B: Ana is wondering how many years the USA has been a nation.	2: Count by ones
C: The height of Mt. Everest.	3: Count by fifty.

Match one scenario and one scale. Explain your thinking. How did you come to this conclusion?

A) _____

_____

_____

B) _____

_____

_____

C) _____

_____

_____

Estimate where you think the following scenarios would fit on one of the numbers lines below. Place a point where you think the point would fit.

    a.   Number of gold fish in a bowl (fish)
    b.   The age of the U.S.A. (years)
    c.   The height of Mount Everest. (feet)

27000   28000   29000   30000

5   10

100   200   300

11
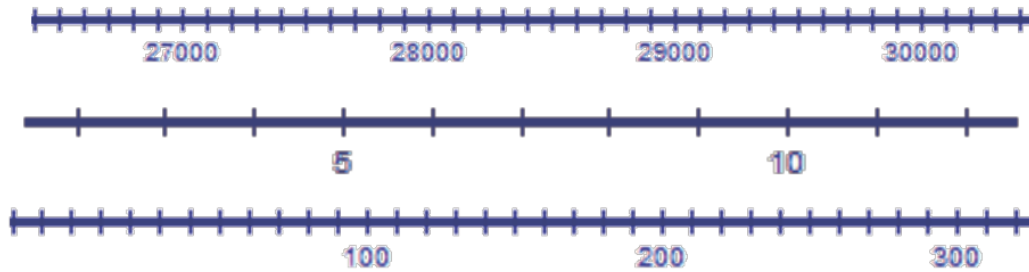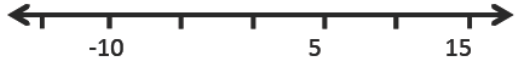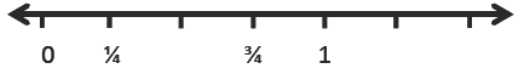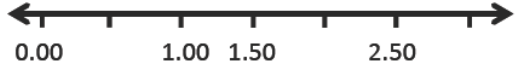
Part 2: Concept, Skill, and Application Exercise

Situation	Scale/Increment	Place numbers on number line
The number of people in a household		
The weight of an elephant		
The cost of a soda		
The winter temperature in Alaska		
The weight of a person		
The number of letters in a person's first name		
The speed of a car		
A person's weekly salary		

Part 3: Concept, Skill, and Application Exercise

Think of a real life situation that you would use for each number line. Fill in the missing numbers.

	1 2
	-10 5 15
	2 4 8
	0 ¼ ¾ 1
	0 2000 4000
	0.00 1.00 1.50 2.50
	-30 -10 -5

This page was intentionally left blank.

3a-2 Representing equations and inequalities

Revisit the problems from yesterday, and represent the solutions on a number line.

Context	Equation or Inequality ex. $x > 8$	Representation
Javier has enough to pay for his lunch at the amusement park. Lunch costs $10. What are the possible amounts that he may have?	$x \geq 10$	
Ms. G wants to take her Avid class to Knott's for their end of the year field trip. It will only happen if more than 20 students pass the math test. How many students need to pass?	$20 < x$	
There has to be more than 20 students but no more than 60 on the bus. How many students can fill one bus?	$20 < x \leq 60$	
Each log on the log ride fits no more than five people. If you have less than five people, then the ride will not go. How many riders must be on log?	$5 = x$	

What patterns did you notice? How do you determine the direction of the shaded line? Discuss these questions with your team and record your responses below.

_____

_____

_____

_____

When graphing solutions onto a number line, the two symbols below are used to represent the **boundary point:**

◯

●

Sometimes the circle is *shaded* and sometimes it is *un-shaded*. What do you think these symbols might represent? What do you think determines whether or not the boundary point is *shaded* or *un-shaded*? Talk with your teams and record your answers below:

_____

_____

_____

_____

Given		*Given*
= "is equal to"		= ●
> "is greater than"	➡	> ◯
< "is less than"		< ◯
≤ "is less than or equal to"		≤ ●
≥ "is greater than or equal to"		≥ ●
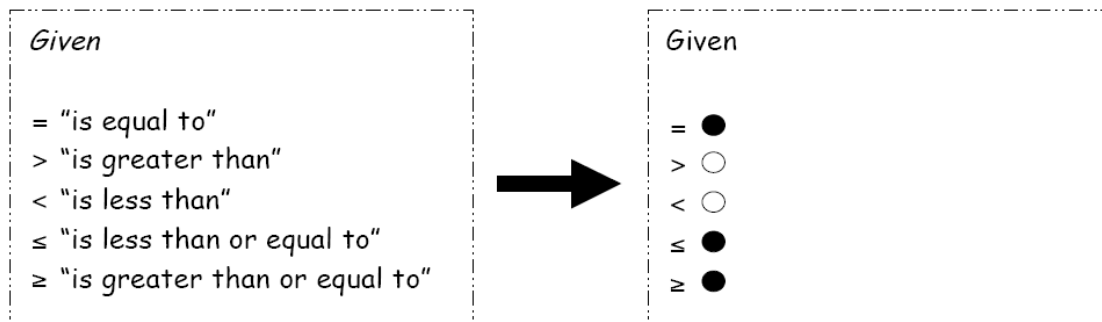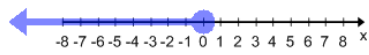
What patterns did you notice? When are the circles *shaded*? When are they *un-shaded*? Feel free to change your number lines on page 5.

_____

_____

_____

_____

Apply your new understanding to the table below:

Context	Equation/Inequality	Representation
Tommy has more than $5 in his pocket.		⟵⟶
		⟵ ─○─⟶ -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 x
	$3 \geq x$	⟵⟶
		⟵ ●─ ⟶ -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 x
	$x = 7$	⟵⟶
To ride Space Mountain, you must be at least 50 inches tall.		⟵⟶
Create your own!		

17

This page was intentionally left blank.

3a-3 Inverse Operations and Solving Equations and Inequalities

Here is a list of things people do every day. Decide which of the activities are things that "un-do" each other.

Juan wakes up and ties his shoes. When he gets home he unties his shoes.

Juan makes a mess in his room, and then he cleans his room.

Rose wrote the answer to a problem, and then erased it.

Lassie dug a hole for his bone in the backyard, and then filled the hole.

A quarterback ran 5 yards, but then got sacked and lost 5 yards.

Todd put $20 in his wallet, later in the day it fell out of his wallet.

Maria filled her tank with gas, drove a long distance, then ran out of gas

Jorge got paid $300 then spent all of it on a new iPod

Greg owed $10 to his uncle then found $10 on the street to give him

The soccer ball was deflated so the coach filled it with air.

The idea of "un-doing" something occurs in mathematics as well. In math, we call these **inverse operations.** Using what you've learned about "un-doing" something, fill in the blanks below:

The **inverse operation** of addition is _____.
The **inverse operation** of subtraction is _____.
Multiplication is the _____ **operation** of division.
The **inverse** _____ of _____ is multiplication.

To undo $+3$, you must _____.
To undo -2, you must _____.
To undo $\times 3$, you must _____.
To undo $\div 7$, you must _____.

Main idea:

_____

_____

Alex and Jocelyn are headed to Knott's on a field trip with Ms. Obtuse. Alex's parents gave him $30 and Jocelyn's parents gave her $25. At 11:30 AM, Alex and Jocelyn grabbed lunch. The bill came to $20 and Alex and Jocelyn decided to split the bill. At 2:00 PM Jocelyn met up with a few of her other friends and spent some money playing games, and Alex and bought souvenirs for his brothers and sisters. At this point, Jocelyn had $5 more than Alex who had $7 in his pocket. Near the end of the day Alex went and bought ice cream, while Jocelyn went to the gift shop. At the end of the day, they met at the front of the park and after counting their money they found that Alex had spent $6 more than Jocelyn. Alex had $5 left when he returned home.

Scratch work:

1. How much did Jocelyn and Alex have at the start of the day?

2. How much money did Jocelyn and Alex have at noon?

3. How much money did Jocelyn and Alex have at 2:00 PM?

4. How much money did Jocelyn and Alex have at the end of the day?

Equations can be used to _____. They can be solved to find _____ value. For example, the equation $x - 3 = 7$ can be used to model the following problem
"_____
_____?"

Inequalities can also be used to _____. They can be solved to find_____. For example, the inequality $x - 3 > 7$ can be used to model the following problem,
"_____?"

Equations and inequalities can be solved using _____.

20

$$x - 3 = 7$$

$$x - 3 > 7$$

Write the equation above in words:

_____
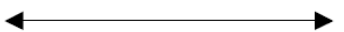
Write the inequality above in words:

_____

Revisit problems 2 through 4 on page 9. Write and solve equations or inequalities for each problem.

2.	3.	4.

Let's Practice!

In words	Equation or Inequality	Representation
Jaime had $4 at the end of the day. She spent $13 on food and groceries. How much did she have at the start of the day?		$\longleftrightarrow$
	$15 > 3x$	$\longleftrightarrow$
Jose has $12. His friend borrowed some money, and now Jose has less than $8.		$\longleftrightarrow$
Three more than a number is nine		$\longleftrightarrow$
Make your own!		$\longleftrightarrow$

Main idea:

_____

_____

3c-1 The Theme Park Ride

1. You and a friend entered a contest and won two tickets to Knott's. Both of you decided that you want to ride on Silver Bullet first.

The **minimum** height requirement is 54 inches and the **maximum** is 84 inches.

Your friend is 62 inches tall. Your height is _____ inches.

Can both of you ride together? _____, because _____

_____

There are many people including little children that want to ride the Silver Bullet. Represent the height restrictions mathematically.

2. For the following guests decide whether or not they meet the height requirements. Represent the information on the given number line below. Label each person's first name initial on the number line.

Guest	Height	Visual Representation
Juan	61 in	
Sarah	70 in	
Christi	55 $\frac{1}{2}$ in	
Alberto	3 ft	
Berry	54 in	
David	73 in	
Nicole	65 in	
Rachel	42 in	
Teresa	5 ft	
Mark	7ft 2in	

min   max

Height (inches)

List all riders who are eligible to ride:

_____

3. Let x represent the heights of all qualified riders:

> What must x be greater than or equal to? $x \geq$ ___
>
> AND
>
> What must x be less than or equal to? $x \leq$ ___

According to the safety rules, **both** height requirements must be met in order to ride.
Create a **compound inequality** to represent the height restriction mathematically.

$$\_\_ \leq x \leq \_\_$$

Why do you think this is called a **compound inequality**?

_____

_____

4. Your task is to create **compound inequalities** for the following situations and represent them on a number line:

a) Today's temperature will reach a high of 70°F and a low of 54°F . ____ $\leq t \leq$ ____

b) In 2012, a household income in the USA making more than $40,000 and less than $95,000 a year was considered middle class.   40,000 m 95,000

c) Assuming there is no traffic on the 405 freeway, the fastest a person can legally drive is 65 miles per hour, and the slowest is 45 mph.   _____

d) According to the FDA, milk should never be warmer than 45°F or cooler than 32°F while storing it.   _____

e) According to salary.org, the average hourly wage without a college degree is between $8 and $15.   _____
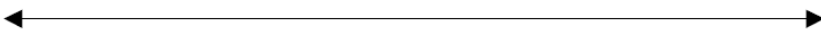
f) Water turns to solid when it reaches 32°F. However it becomes a gas when it reaches 212°F. Write a compound inequality for when water is a liquid.   _____
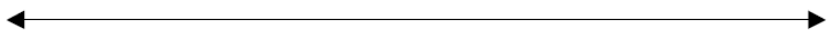
5. Consider your original hypothesis of what a **compound inequality** is (from page 12) and the definition at the top of this page. Write a *new* definition of a **compound inequality** in your own words:

_____

_____

Refer back to examples 4a to 4f. What do all number lines have in common?

_____

24

6. Create a compound inequality to represent the heights of the guest who were not allowed to ride on the Silver Bullet.

Guests are not allowed to ride the Silver Bullet if they are less than _____ inches, $x < $ ____

OR

Guests are not allowed to ride the Silver Bullet if they are more than _____inches, $x > $ ____

Represent the guests that **are not allowed** to ride graphically on a number line.

The **compound** _____ that represents the guests who **can** ride was _____.

The **compound** _____ that represents the guests who **can't** ride is: $x < $ ____ or $x > $ ____

Definition: A compound inequality is two inequalities joined by the word _and_ or the word _or_.

---

7. **Your task** is to create **compound** _____ and represent them on a number line:

a) Write a _____ **inequality** for the temperatures when water in **not** a liquid, and then graph your answer below. $x \leq $ ____   or $x \geq $ ____
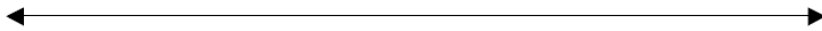
b) Describe the temperature that you should not store milk at mathematically and graphically.

c) Write a compound inequality that would describe unlikely air temperatures in Orange County, CA. Represent it on the number line.

d) Refer back to problem 4c. Write a _____ **inequality** for a situation when you **could** get a ticket.

25
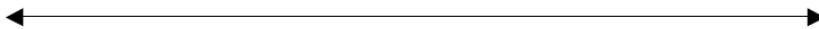
8. Compare and contrast the two types of **compound inequalities** using a diagram of your choice.

9. Use your own words to describe the difference between compound inequalities with **_and_** and **_or._**

_____
_____
_____
_____

10. Create two stories and represent them with compound inequalities and graphs.

and	*or*

3c-2 Simultaneous Equations and Inequalities

1. The mayor of Santa Ana is going on a business trip to Russia. His secretary checked that the average temperature in May in Moscow, Russia is 10°C. His secretary also gave him the formula to convert °C to °F.

The formula is $F = \frac{9}{5}C + 32$.

The secretary is out sick so your task is to convert the average temperature in May in Fahrenheit and report to the mayor.

Use the following table below to plan and organize your work. **Answer in complete sentences**.

What do you know?	
What are we looking for?	
How many equations are there? What are they?	
What do you notice is the same in each equation? What could you do with the equations?	

Mathematical Representation of the Problem

$$\begin{cases} C = \underline{\quad} \\ F = -C + \underline{\quad} \end{cases}$$

$$F = - (\quad) + \underline{\quad}$$

What will you report to the mayor?

The mayor of Santa Ana is pleased with your work and has decided to hire you. Your new task is to find the average temperature in Celsius for Santa Ana. The average daily temperature for the month of May in Santa Ana is 77°F.

What do you know?	
What are we looking for?	
How many equations are there? What are they?	
What do you notice is the same in each equation? What could you do with the equations?	
Mathematical Representation of the Problem $\begin{cases} F = \underline{\quad} \\ F = -C + \underline{\quad} \end{cases}$ 	

What will you report to the mayor?

Definition: A system is two or more equations or inequalities represented by the brace symbol, $\{$

For the next two examples use the formula (equation) that converts Celsius to Fahrenheit to create and solve a system of equations.

1. The boiling point of water is 100°C. Find the boiling point of water in degrees Fahrenheit.

$$\begin{cases} C = \underline{\hspace{1cm}} \\ F = -C + \underline{\hspace{0.5cm}} \end{cases}$$

2. Water freezes at 32°F. Find the freezing temperature in degrees Celsius.

$\{$

3. Solve the following system of equations

$$\begin{cases} 3x + 2y = 8 \\ x = 7 \end{cases}$$

4. Solve the following system of one inequality and one equation

$$\begin{cases} 3x - 4y \leq 24 \\ y = 3 \end{cases}$$

Looking back to the definition box and examples 1 to 4, redefine systems of equations (inequalities) in your own words.

_____

_____

What is the symbol used to group the equations (inequalities) of a system?

This page was intentionally left blank.

3c-3 Simple Interest

The following text is from Wikipedia.org about **simple interest**. Read and analyze the text before answering the questions below.

Interest is a fee paid by a borrower of assets to the owner as a form of compensation for the use of the assets. It is most commonly the price paid for the use of borrowed money,[1] or money earned by deposited funds.[2]

When money is borrowed, interest is typically paid to the lender as a percentage of the principal, the amount owed to the lender. The percentage of the principal that is paid as a fee over a certain period of time (typically one month or year) is called the interest rate. A bank deposit will earn interest because the bank is paying for the use of the deposited funds. Assets that are sometimes lent with interest include money, shares, consumer goods through hire purchase, major assets such as aircraft, and even entire factories in finance lease arrangements. The interest is calculated upon the value of the assets in the same manner as upon money.

Simple interest

Simple interest is calculated only on the principal amount, or on that portion of the principal amount that remains unpaid.

The amount of simple interest is calculated according to the following formula:

$$I_{simp} = r \cdot B_0 \cdot m_t$$

where r is the period interest rate (I/m), B_0 the initial balance and m_t the number of time periods elapsed.

To calculate the period interest rate r, one divides the interest rate I by the number of periods m_t.

1. What does the letter I in the formula represent?

2. What does the letter r in the formula represent?

3. What does the letter B_0 in the formula represent?

4. What does the letter m_t in the formula represent?

31

1. Uncle Sam's family has an outstanding credit card balance of $5,000. His credit card has an annual simple rate of 18%. If he did not make any payments for 3 years, how much interest would he owe? What will the new balance be?

Create and solve a system of equations to represent this problem.

$$\begin{cases} I = \\ r = \\ m_t = \\ B_0 = \end{cases}$$

2. You are off to college and you need a car to get to class and work. You decided to borrow money from the bank at a simple interest rate of 8% for 4 years. The car you bought costs $_____

Calculate the interest rate on the loan.

3. While in college you bought a computer that cost $1,400. You bought the computer using your parent's credit card when you were 18 years old. On your 28th birthday your parents tell you that you owe them $2,240.

How much interest (in dollars) has been added to the original cost?

How much time has passed (in years)?

Create and solve a system of equations to find the interest rate:

This page was intentionally left blank.

Summative Assessment   **Name** _____

Direction: You are to work on this Assessment with a partner.

Fishing Adventures rents small fishing boats to tourists for day long fishing trips. There are two options for boat rentals that you need to consider in order to choose the appropriate boat for your group.

Option 1: (Adults only)

Each boat can hold at most 1200 pounds of people and gear for safety reasons. Assume on average an adult weights 150 pounds and are allowed to have 15 pounds of gear each. Also assume each group will require 200 pounds of gear.

Question: How many adults are allowed on the boat? Illustrate your reasoning algebraically and graphically by providing

   a. An inequality that represents the weight limit and the total of passengers allowed on the boat
   b. A solution set to the inequality on a number line or coordinate plane.

Option 2: (Family)

Each boat can hold at most eight people. Additionally, each boat can only carry 1200 pounds of people and gear for safety reasons. Assume on average an adult weighs 150 pounds and a child weighs 75 pounds. Also assume each group will require 200 pounds of gear plus 10 pounds of gear per person.

Question: How many adults and children are allowed on this boat in order to optimize the weight limit? Illustrate your reasoning algebraically and graphically by providing

   a. An inequality or system of inequalities that represent the weight limit and the total number of passengers allowed on the boat.
   b. A solution set to the inequalities on coordinate plane.

Question: Compare the two options.

   a. How are these two options different or alike mathematically and graphically?
   b. How was your approach to one option different from the other? Explain.
   c. If you were to pick one option for your family, which option will you go for? Explain.

This page was intentionally left blank.

This page was intentionally left blank.

This page was intentionally left blank.